

TIPS4RM

Targeted Implementation
and Planning Supports for
Revised Mathematics

Continuum and Connections

Proportional Reasoning

Overview

Context Connections

- Positions proportional reasoning in a larger context and shows connections to everyday situations, careers, and tasks
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers
- Includes relevant specific expectations for each grade
- Summarizes prior and future learning

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied
- Includes advice on how to help students develop understanding

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (proportional reasoning) as an organizer

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question.*
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated
- Serves as a model for developing other questions relevant to the learning expected for the grade/course

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate
- Focuses on problem-solving strategies, involving multiple mathematical processes
- Provides an opportunity for students to engage with the problem at many levels
- Provides problems appropriate for students in Grades 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?

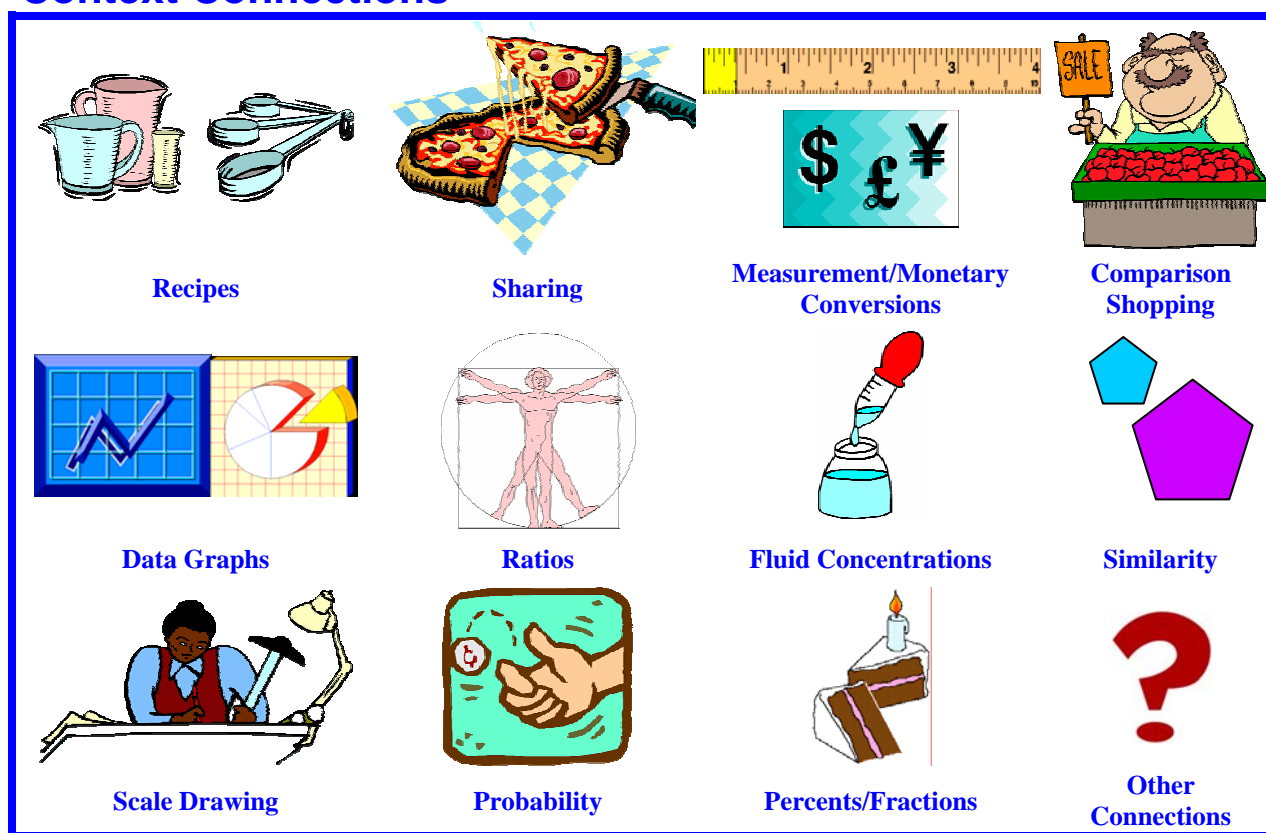
- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical process Reasoning and Proving, and Reflecting

Proportional Reasoning

Context

- Proportional reasoning has been referred to as the capstone of the elementary curriculum and cornerstone of algebra and beyond. (Lesh, Post, & Behr, 1987)
- Students use ratios to describe proportional relationships involving number, geometry, measurement, and probability.
- Students understand and use ratios and proportions to represent quantitative relationships. (NCTM Standards)
- Students develop, analyse, and explain methods for solving problems involving proportions such as scaling, and finding equivalent ratios and rates.
- Proportional reasoning is the ability to think about and compare multiplicative relationships between quantities.

Context Connections



Manipulatives

- coloured tiles
- cubes/geoboards
- fraction circles/rings/rods
- grid paper
- pattern blocks

Technology

- The Geometer's Sketchpad®4
- Fathom®
- calculators/graphing calculators
- spreadsheet software
- virtual manipulatives

Other Resources

<http://mathforum.org/mathtools/cell/m7,9.10.5,ALL,ALL/>
http://www.learner.org/channel/workshops/math/work_6.html
<http://matti.usu.edu/nlvm/nav/vlibrary.html>
<http://math.rice.edu/~lanius/proportions/rate9.html>

Connections Across Grades

Selected results of word search using the Ontario Curriculum Unit Planner

Search Words: *develop, understand, comparison, relationships, investigation, percent, ratio, rate, proportion, proportional reasoning, similar, similarity*

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
<ul style="list-style-type: none"> represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation; determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions, decimal numbers, and percents; represent relationships using unit rates. 	<ul style="list-style-type: none"> determine, through investigation, the relationships among fractions, decimals, percents, and ratios; solve problems that involve determining whole number percents, using a variety of tools; demonstrate an understanding of rate as a comparison, or ratio, of two measurements with different units; solve problems involving the calculation of unit rates. 	<ul style="list-style-type: none"> identify and describe real-life situations involving two quantities that are directly proportional; solve problems involving proportions, using concrete materials, drawings, and variables; solve problems involving percent that arise from real-life contexts; solve problems involving rates. 	<p><u>Applied</u></p> <ul style="list-style-type: none"> illustrate equivalent ratios, using a variety of tools; represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations; solve for the unknown value in a proportion, using a variety of methods; make comparisons using unit rates; solve problems involving ratios, rates, and directly proportional relationships in various contexts, using a variety of methods; solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms. <p><u>Academic</u></p> <ul style="list-style-type: none"> solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion. 	<p><u>Applied</u></p> <ul style="list-style-type: none"> verify, through investigation, properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides); determine the lengths of sides of similar triangles, using proportional reasoning; solve problems involving similar triangles in realistic situations.

Summary of Prior Learning

In earlier years, students:

- build an understanding of simple multiplicative relationships involving whole-number rates, through investigation, using concrete materials and drawings;
- determine and explain the relationship between fractions and their equivalent decimal and percentage amounts, using concrete materials;
- use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;
- represent ratios;
- demonstrate an understanding of relationships involving percent, ratio, and unit rate;
- solve problems requiring conversion from larger to smaller metric units.

In Grade 7, students:

- continue to determine the relationship between fractions, decimals, and percent;
- develop an understanding of rates as a comparison of two measures with different units;
- solve problems involving whole number percents and unit rates using a variety of tools and strategies;
- determine conversions of metric units of measure;
- demonstrate an understanding of proportional relationships using percent, ratio, and rate.

In Grade 8, students:

- continue to solve problems that involve percent and rates;
- identify and solve problems involving proportions, using concrete materials;
- solve problems using proportional reasoning in meaningful contexts;
- identify and describe real-life situations involving quantities that are directly proportional.

In Grade 9 Applied, students:

- solve for an unknown in a proportional relationship by using a variety of methods including algebraic methods;
- solve problems that require a fraction, decimal, or percent to be rewritten in an equivalent form;
- make comparisons using unit rates;
- represent and solve problems involving ratios, rates, and directly proportional relationships in context.

In Grade 10 Applied, students:

- use their knowledge of ratios and proportions to investigate and use the property of similar triangles that the ratios of corresponding sides are equal;
- solve problems involving similar triangles in realistic situations.

In later years

Students' choice of courses will determine the degree to which they apply their understanding of concepts related to proportional reasoning.

Instruction Connections

Suggested Instructional Strategies	Helping To Develop Understanding
<p>Grade 7</p> <ul style="list-style-type: none"> Investigate and represent the relationships among fractions, decimals, and percents, using concrete materials. Connect students' knowledge about percents to make comparisons in context. Express probabilities as ratios. Apply both experimental and theoretical probabilities to make predictions. Investigate dilatations, using a variety of tools, e.g., pattern blocks, overhead transparencies, computer technology. Create similar triangles on the Cartesian plane using grid paper, measure and compare angles, longest, shortest, and remaining sides. Enlarge/reduce similar triangles to build understanding of the proportionality of similar triangles, using The Geometer's Sketchpad®4. Model rates by graphing to visualize the relationship, and by creating algebraic expressions. Illustrate ratios, using scale drawings. Model situations requiring the determination of best rates, using geoboards. <p>Grade 8</p> <ul style="list-style-type: none"> Investigate the relationship between the circumference and the diameter of a circle as a ratio to determine π. Solve percent problems arising from everyday familiar contexts in more than one way. Solve problems involving proportions, using concrete materials and drawings. Develop an understanding that proportions are multiplicative relationships. Represent probabilities in multiple ways, e.g., fractions, percents, ratios. Investigate similar figures using concrete materials and The Geometer's Sketchpad®4. Investigate proportional relationships among measurable attributes of geometric figures. <p>Grade 9 Applied</p> <ul style="list-style-type: none"> Explore and develop an understanding of proportions, using a variety of contexts, tools, and methods. Investigate ratios using examples of proportional and non-proportional situations to arrive at an understanding of the multiplicative relationship of proportions. Investigate a variety of methods for solving problems involving proportions, e.g., ratio tables, drawings, graphs, unit rates, scaling factors. Support students in developing proportional reasoning, and procedural fluency in its application. <p>Grade 10 Applied</p> <ul style="list-style-type: none"> Investigate the properties of similar figures, using concrete materials, e.g., geoboards, The Geometer's Sketchpad®4. Convert imperial measurement and metric measurement, using proportions. Solve problems involving similar triangles in familiar contexts, using a variety of methods. 	<ul style="list-style-type: none"> When examining two ratios, it is sometimes useful to think of them as being either <i>within</i> ratios or <i>between</i> ratios. A ratio of two measures in the same setting is a <i>within</i> ratio. A <i>between</i> ratio is a ratio of two corresponding measures in different situations. (John Van de Walle) Provide opportunities for students to think about ratios in these two ways. A ratio can compare a part to a whole (part-to-whole) or a part of the whole to another part of the whole (part-to-part). Provide opportunities for students to develop their understanding by setting activities in a wide range of contexts. Demonstrate that ratios can also compare measures of two different types. (rates) Relate proportional reasoning to existing processes, for instance the concept of unit fractions is similar to unit rates. Delay the use of rules and algorithms for solving proportions, until students have developed proportional reasoning skills. Encourage discussion and experimentation in comparing ratios. Vary the numerical relationships or the context in which the problems are posed. Pose problems for students to explore that help them to identify multiplicative situations. Fraction, ratio, and rational number ideas are mathematically complex and interconnected. Provide students with time to construct important ideas and ways of thinking. (Lamon) Give students multiple opportunities to progress through different representations – concrete → diagrams/tables → symbolic. Have students write word statements to describe their reasoning. Have students represent proportion problems as unit rates. Use numerical comparison problems where two complete rates are given and students are asked to answer questions about how the rates are to be compared. Present problems that involve qualitative prediction and comparison that require responses not dependent on specific numerical values. Research suggests teaching multiple strategies, starting with unit rate and factor of change which are more intuitive then followed by fractions and the cross product algorithm.

Connections Across Strands

Note

Summary or synthesis of curriculum expectations is in plain font.

Verbatim curriculum expectations are in italics, but may not include the examples or phrases.

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
<ul style="list-style-type: none"> <i>determine, through investigation, the relationships among fractions, decimals, percents, and ratios</i> <i>solve problems that involve determining whole number percents, using a variety of tools</i> <i>demonstrate an understanding of rate as a comparison, or ratio, of two measurements with different units</i> <i>solve problems involving the calculation of unit rates</i> 	<ul style="list-style-type: none"> <i>solve problems that require conversion between metric units of measure</i> <i>solve problems that require conversion between metric units of area</i> 	<ul style="list-style-type: none"> <i>construct angle bisectors and perpendicular bisectors, using a variety of tools and strategies, and represent equal angles and equal lengths using mathematical notation</i> <i>demonstrate an understanding that enlarging or reducing two-dimensional shapes creates similar shapes</i> <i>identify, perform, and describe dilatations through investigation using a variety of tools</i> 	<ul style="list-style-type: none"> <i>represent linear growing patterns, using a variety of tools and strategies (for a proportional relationship)</i> <i>make predictions about linear growing patterns, through investigation with concrete materials</i> <i>model real-life relationships involving constant rates where the initial condition starts at 0, through investigation using tables of values and graphs</i> <i>model real-life relationships involving constant rates, using algebraic equations with variables to represent the changing quantities in the relationship</i> <i>solve linear equations of the type $ax = c$ or $c = ax$</i> 	<ul style="list-style-type: none"> <i>research and report on real-world applications of probabilities expressed in fraction, decimal, and percent form</i> <i>make predictions about a population when given a probability</i> <i>determine the theoretical probability of a specific outcome involving two independent events</i>

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
<ul style="list-style-type: none"> • identify and describe real-life situations involving two quantities that are directly proportional • solve problems involving proportions, using concrete materials, drawings, and variables • solve problems involving percent that arise from real-life contexts • solve problems involving rates • translate between equivalent forms of a number (i.e., decimals, fractions, percents) 	<ul style="list-style-type: none"> • solve problems that require conversions involving metric units of area, volume, and capacity • determine, through investigation using a variety of tools and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas 	<ul style="list-style-type: none"> • determine, through investigation using a variety of tools, relationships among area, perimeter, corresponding side lengths, and corresponding angles of similar shapes 	<ul style="list-style-type: none"> • represent patterns using an algebraic expression • model a proportional relationship using tables of values, graphs, and equations 	<ul style="list-style-type: none"> • determine the theoretical and experimental probability of an event occurring or not occurring • compare two attributes or characteristics using a scatter plot and determine whether or not the scatter plot suggests a relationship • compare two attributes or characteristics, using a variety of data management tools and strategies

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relations
<ul style="list-style-type: none"> • illustrate equivalent ratios, using a variety of tools • represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations • solve for the unknown value in a proportion, using a variety of methods • make comparisons using unit rates • solve problems involving ratios, rates, and directly proportional relationships in various contexts, using a variety of methods • solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms 	<ul style="list-style-type: none"> • determine, through investigation, that the rate of change of a linear relation can be found by choosing any two points on the line that represents the relation, finding the vertical change between the points (i.e., the rise) and the horizontal change between the points (i.e., the run), and writing the ratio $\frac{\text{rise}}{\text{run}}$ (i.e., the rate of change = $\frac{\text{rise}}{\text{run}}$) • compare the properties of direct variation and partial variation in applications, and identify the initial value • describe the meaning of the rate of change and the initial value for a linear relation arising from a realistic situation 	<ul style="list-style-type: none"> • develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere • solve problems involving the volumes of prisms, pyramids, cylinders cones, and spheres

Grade 10 Applied

Measurement and Trigonometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
<ul style="list-style-type: none"> • verify, through investigation, properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides) • determine the lengths of sides of similar triangles, using proportional reasoning • solve problems involving similar triangles in realistic situations • determine, through investigation, the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios • perform everyday conversions between the imperial system and the metric system and within these systems, as necessary to solve problems involving measurement 	<ul style="list-style-type: none"> • connect the rate of change of a linear relation to the slope of the line, and define the slope as the ratio $m = \frac{\text{rise}}{\text{run}}$ • identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$ • identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism), using graphing technology to facilitate investigations, where appropriate 	<ul style="list-style-type: none"> • students may realize through investigation that a quadratic relation of the form $y = ax^2$ is a dilatation of the graph of $y = x^2$

Name:

Date:

Expectation – Number Sense and Numeration, 7m30:
Solve problems involving the calculation of unit rates.

Knowledge and Understanding (Facts and Procedures)

Oranges can be purchased at a cost of 3 for \$1.99. What is the cost for 12 oranges?

Show your work.

Knowledge and Understanding (Conceptual Understanding)

An athlete ran 5 km in 35 minutes. What is her rate in km/hr?

Show your work.



Problem Solving (Connecting)

Karen paid \$0.90 per litre for gasoline. If she paid \$27.00 for the gas, how many litres did she purchase?

Show your work.



Problem Solving (Reasoning and Proving, Selecting Tools and Computational Strategies)

The following are resting heart rates:

Animal	Resting Heart Rate
Lion	40 beats in 60 seconds
Giraffe	5 beats in 12 seconds
Hummingbird	41 beats in 10 seconds

Which animal has the slowest heart rate?

Show your work and explain your reasoning.

Name:

Date:

Expectation – Number Sense and Numeration, 8m27:

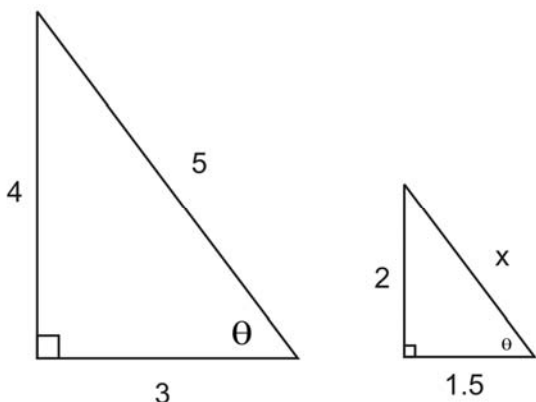
Solve problems involving proportions, using concrete materials, drawings, and variables.

Knowledge and Understanding

(Facts and Procedures)

The triangles shown are similar. Solve for the missing side.

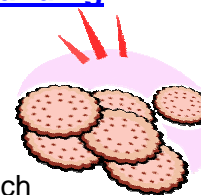
Show your work.



Knowledge and Understanding

(Conceptual Understanding)

A recipe for 4 dozen shortbread cookies uses 500 g flour, 450 g of butter, and 200 g of sugar. How much of each ingredient would be required to make 60 cookies?



Show your work.

Problem Solving

(Representing and Reflecting)

Use connecting cubes to construct two different models of a building that is 42 metres high, 36 metres wide, and 24 metres long.

What scale would require the fewest number of cubes? Explain.

Show your work.

Problem Solving

(Connecting, Reasoning and Proving)

For their Grade 8 graduation, students are preparing a fruit punch made up of one part orange juice to two parts lemonade. If they purchased 27 cans of beverage, how many were orange juice and how many were lemonade?



Show your work and explain your reasoning.

Name:

Date:

Expectation – Number Sense and Algebra, NA1.05: Solve problems involving ratios, rates, and directly proportional relationships in various contexts, using a variety of methods.

Knowledge and Understanding (Facts and Procedures)

The Canadian dollar is worth 87 cents American. If you purchased a \$50 item in the United States, how much would it cost in Canadian funds?



Show your work.

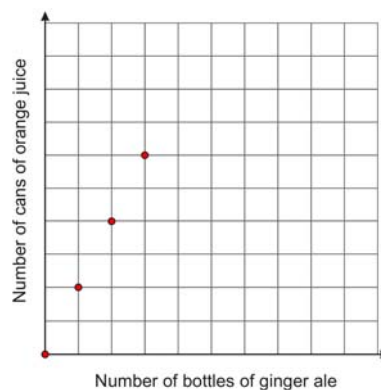
Knowledge and Understanding (Conceptual Understanding)

A standard punch requires two cans of orange juice and a bottle of ginger ale. When using 4 bottles of ginger ale, how many cans of orange juice are needed for the punch?

Use the graph to solve the problem.

Show your work.

Standard Punch



Problem Solving (Representing and Reflecting)

In a survey, 9 out of the 28 students in a Grade 9 class have part time jobs. In a school of 196 students in Grade 9, there are 63 with part time jobs. Does this answer make sense to you?

Explain.

Show your work.

Problem Solving (Connecting, Reasoning and Proving)

On a walk Anne takes 3 steps for every 7 that Rod takes. How many steps has Anne taken when Rod has taken 42 steps?

Use two methods to solve the problem.

Show your work and explain your reasoning.

Name:

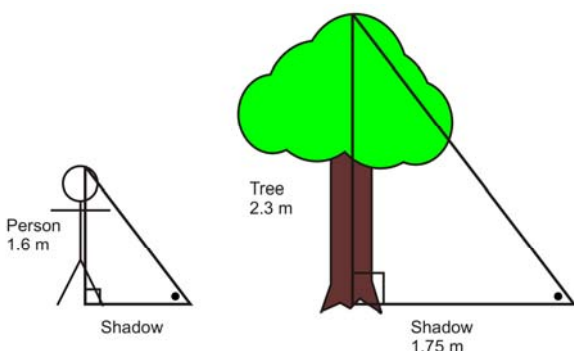
Date:

Expectation – Measurement and Trigonometry, MT1.02: Determine the lengths of sides of similar triangles using proportional reasoning.

Knowledge and Understanding (Facts and Procedures)

Solve for the length of the shadow of the man.

Show your work.

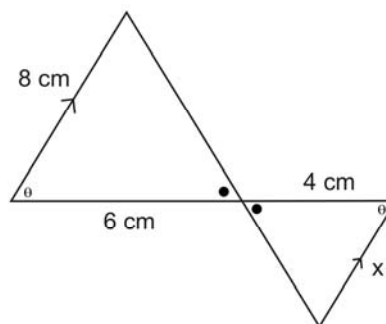


Hint: the triangles are similar.

Knowledge and Understanding (Conceptual Understanding, Facts and Procedures)

A surveyor took the measurements shown in the diagram. Determine the measure of x .

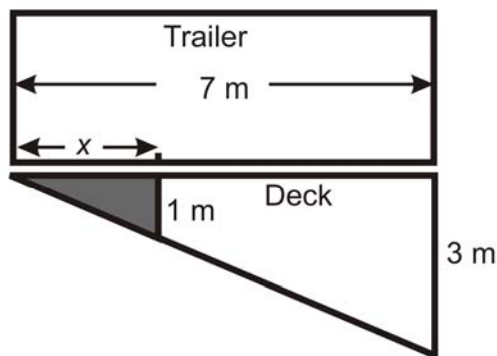
Show your work.



Problem Solving (Connecting and Reflecting)

A 7-metre trailer has a triangular deck. The shaded triangle represents the area for the BBQ. Find the length of the side of the shaded area that is adjacent to the trailer.

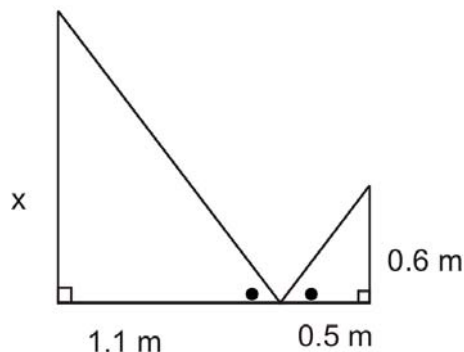
Show your work.



Problem Solving (Connecting and Reflecting)

Pose a problem that could be solved using the given diagram of similar triangles.

Show your work and explain your reasoning.



Problem Solving Across the Grades

Sample 1

Name:

Date:

A local recreation centre is building a new outdoor pool to accommodate the growing population. The old pool contained 120 000 gallons of water, and required 50 pucks of chlorine per week. The new pool will contain 500 000 gallons of water. How many pucks of chlorine will be required to chlorinate the new pool for a week?

1.

2.

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1.

120 000 gallons of water \div 50 pucks = 2400 gallons of water for 1 puck
 500 000 gallons of water \div 2400 gallons/puck = 208.33 pucks

Problem-Solving Strategies:

- Use unit rates

It could take 208 or 209 pucks.

2.

Number of Gallons of Water	Number of Pucks	Comments
120 000	50	
240 000	100	double
480 000	200	double again
2400	1	$\frac{240\,000}{100}$
24 000	10	2400×10
4800	2	2400×2

Problem-Solving Strategies:

- Use a ratio table
- Use estimation

Estimate:

$$500\,000 \div 480\,000 + 24\,000 - 4800$$

$$\text{or } \div 499200$$

$$500\,000 \div 480\,000 + 24\,000 - 2400 \div 501\,600$$

Therefore, the number of pucks needed is between $200 + 10 - 2$ and $200 + 10 - 1$.

So, the number of pucks needed is between 208 and 209.

(Round up to 209 pucks for safety.)

Students' solutions could include any of the Grade 7 answers.

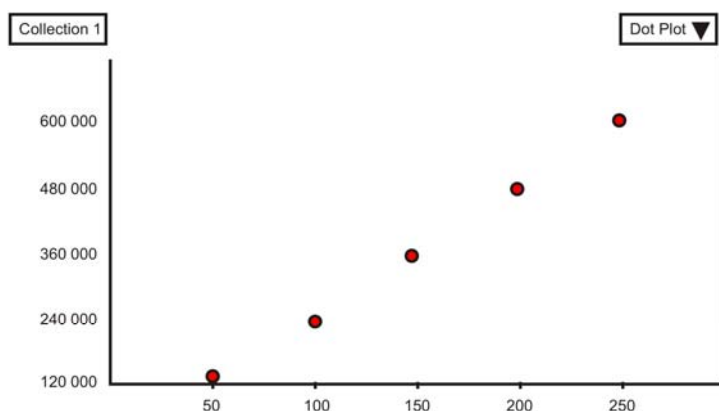
1.

Gallons of Water	Pucks
120 000	50
240 000	100
360 000	150
480 000	200
600 000	250

Problem-Solving Strategies:

- Make an organized list
- Guess and check
- Use a graph

500 000 gallons of water is between 480 000 gallons and 600 000 gallons. Therefore, between 200 and 250 pucks would be needed.



2. a) “Between” ratios

120 000 is proportional to 500 000.

The scale factor is $500\,000 \div 120\,000$

$$= \frac{50}{12}$$

$$50 \times 4 \frac{1}{6} = 200 + \frac{50}{6} = 208 \frac{1}{3}$$

$$= 4 \frac{1}{6} \quad (\div 4.166667)$$

Problem-Solving Strategies:

- Use a scale factor
- Use “between” or “within” ratios

b) “Within” ratios

$$\frac{5}{12\,000} = \frac{x}{500\,000}$$

$$12\,000x = 5(500\,000)$$

$$x = \frac{2500}{12} \div 208.33$$

They would use 209 pucks.

Students' solutions could include any of the Grades 7 and 8 answers.

1.
$$\frac{\text{chlorine pucks in the old pool}}{\text{gallons of water in the old pool}} = \frac{\text{chlorine pucks in the new pool}}{\text{gallons of water in the new pool}}$$

$$\frac{50 \text{ pucks}}{120\,000 \text{ gallons}} = \frac{x}{500\,000 \text{ gallons}}$$

First multiply both sides by 500 000 gallons.

$$(500\,000 \text{ gallons}) \frac{50 \text{ pucks}}{120\,000 \text{ gallons}} = \frac{(500\,000 \text{ gallons})(x)}{500\,000 \text{ gallons}}$$

Use a calculator to compute the left-hand side of the equation.

$$208.333333 \text{ pucks} = x$$

It would take 209 pucks to keep the pool chlorinated.

Problem-Solving Strategies:

- Use proportion (equivalent ratios)
- Use a calculator

2.

$$m = \frac{240\,000 - 120\,000}{100 - 50}$$

$$= \frac{1200}{5}$$

$$y = 2400x$$

$$2400x = 500\,000$$

$$x = \frac{500\,000}{2400}$$

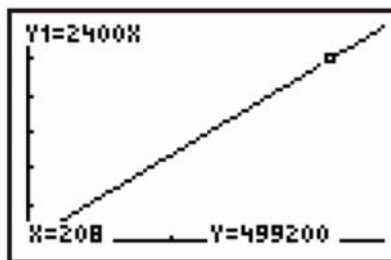
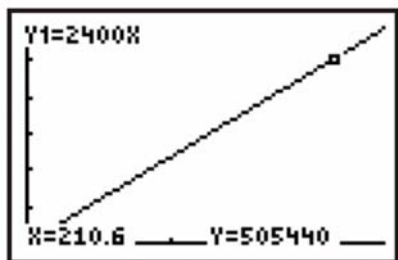
$$= 208.\bar{3}$$

It will take 209 pucks.

Problem-Solving Strategies:

- Use equation of line
- Use a chart and graph

3.



Problem-Solving Strategies:

- Use technology (graphing calculator)

It will take about 209 pucks.

Problem Solving Across the Grades

Sample 2

Name:

Date:

A trip is being planned from Kingston to Toronto. By car it is a 200 km drive and by train the route is 250 km. If the car averages 90 km/h and the train averages 120 km/h, which means of travelling is faster? Show at least two solutions.

1.

2.

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1.

Car		Train	
Hours	Kilometres	Hours	Kilometres
1	90	1	120
2	180	2	240
3	270	3	360

Problem-Solving Strategies:

- Make an organized list or ratio table
- Use logic and reasoning

Students note that after 2 hours the car has 20 km to go, and the train only has 10 km to go. Since the train is going faster and has less distance left after 2 hours, it would get there first.

2.

The car:

$$\frac{90 \text{ km}}{1 \text{ hour}} = \frac{200 \text{ km}}{x}$$

Students see what 90 is multiplied by to get to 200.
(200 divided by 90 = 2.22)

They then multiply the denominator by the same factor to get the equivalent ratio. (1×2.22)

So, the car takes 2.22 hours

Problem-Solving Strategies:

- Use a formula

The train:

$$\frac{120 \text{ km}}{1 \text{ hour}} = \frac{250 \text{ km}}{x}$$

Students see what 120 is multiplied by to get to 250.
(250 divided by 120 = 2.08)

They then multiply the denominator by the same factor. $(1 \times 2.08 = 2.08)$

So, the train takes 2.08 hours

Grade 7

Sample Solutions

3.

Car

90 km/hour
90 km/60 minutes
15 km/10 minutes

Minutes	Kilometres
10	15
20	30
30	45
40	60
50	75
60	90
70	105
80	120
90	135
100	150
110	165
120	180
130	195
140	210
The car travelled 200 km between 130 minutes and 140 minutes.	

Train

120 km/hour
120 km/60 minutes
20 km/10 minutes

Minutes	Kilometres
10	20
20	40
30	60
40	80
50	100
60	120
70	140
80	160
90	180
100	200
110	220
120	240
130	260
The train travelled 250 km between 120 minutes and 130 minutes.	

Problem-Solving Strategies:

- Make an organized list

Therefore, the train took less time and arrived first.

Grade 8

Sample Solutions

Students' solutions could include any of the Grade 7 answers.

Car

$$\frac{90 \text{ km}}{1 \text{ hour}} = \frac{200 \text{ km}}{x \text{ hours}}$$

Train

$$\frac{120 \text{ km}}{1 \text{ hour}} = \frac{250 \text{ km}}{x \text{ hours}}$$

Problem-Solving Strategies:

- Use a formula

For the car, to get from 90 down to 1, divide by 90. On the right hand side 200 divided by 90 equals 2.22 hours to complete the journey by car. For the train to get from 120 down to 1, divide by 120. On the right hand side also divide by 120. The train takes 250 divided by 120 or approximately 2.08 hours.

Students' solutions could include any of the Grades 7 and 8 answers.

a) Car Travelling Time (using “between” ratios)

$$\frac{\text{car distance}}{\text{car time}} = \frac{\text{total distance}}{\text{total time}}$$

$$\frac{90 \text{ km}}{1 \text{ hour}} = \frac{200 \text{ km}}{x}$$

Solve by first multiplying both sides by x .

$$(x) \frac{90 \text{ km}}{1 \text{ hour}} = \frac{200 \text{ km}}{x} (x)$$

$$(x) 90 \text{ km/hour} = 200 \text{ km}$$

Divide both sides by 90 km/hour.

$$\frac{(x) 90 \text{ km/hour}}{90 \text{ km/hour}} = \frac{200 \text{ km}}{90 \text{ km/hour}}$$

$$x = 2.22 \text{ hours}$$

Problem-Solving Strategies:

- Use proportion (equivalent ratios)
- Use “between” and “within” ratios

Train Travelling Time

$$\frac{\text{train distance}}{\text{train time}} = \frac{\text{total distance}}{\text{total time}}$$

$$\frac{120 \text{ km}}{1 \text{ hour}} = \frac{250 \text{ km}}{x}$$

Solve by first multiplying both sides by x .

$$(x) \frac{120 \text{ km}}{1 \text{ hour}} = \frac{250 \text{ km}}{x} (x)$$

$$(x) 120 \text{ km/hour} = 250 \text{ km}$$

Divide both sides by 120 km/hour.

$$\frac{(x) 120 \text{ km/hour}}{120 \text{ km/hour}} = \frac{250 \text{ km}}{120 \text{ km/hour}}$$

$$x = 2.08 \text{ hours}$$

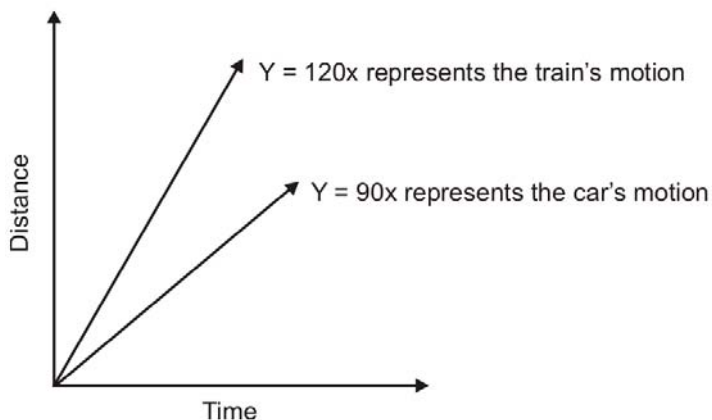
b) As well, the proportion could have been set up using “within” ratios.

$$\frac{\text{car distance}}{\text{total distance}} = \frac{\text{car time}}{\text{total time}}$$

$$\frac{\text{train distance}}{\text{total distance}} = \frac{\text{train time}}{\text{total time}}$$

Students' solutions could include any of the Grades 7, 8, and 9 answers.

a)



Problem-Solving Strategies:

- Use equations and graphs
- Use a table of values
- Use substitution

Using the graph, students read the appropriate points and compare them to demonstrate that the train arrives first.

b)

x (Time in Hours)	Y_1 (Car Distance)	Y_2 (Train Distance)
0	0	0
1	90	120
2	180	240
3	270	360

After 2 hours the car still has 20 km to travel, while the train has 10 km to travel (and is travelling faster than the car). Therefore, the train will arrive first.

c)

$$200 = 90x$$

$$\frac{200}{90} = x$$

$$2.22 = x$$

$$250 = 120x$$

$$\frac{250}{120} = x$$

$$2.08 = x$$

Problem Solving Across the Grades

Sample 3

Name:

Date:

A community is planning to build a new library. The library is being built next to the community pool with 72 metres between them. The builders want to make sure that the library building does not put a shadow over the pool. At the time of the maximum shadow, a flag pole that is 3 metres high gives an 8 metre shadow. What is the maximum height for the library building?

Verify your solution using a different strategy.

1.

2.

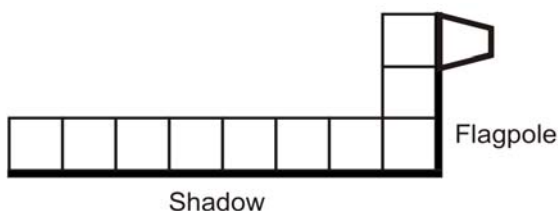
Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1. Students build an “L” shape which is 8 by 3 units, using the edge of the cubes to represent the flagpole and its shadow.

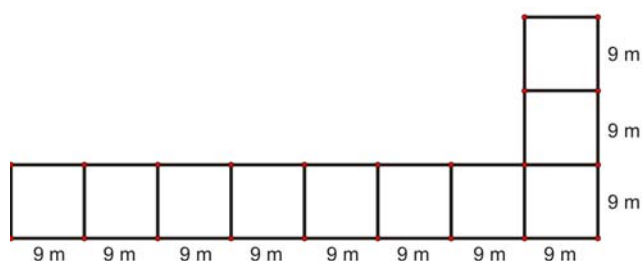
Problem-Solving Strategies:

- Make a scale model, using manipulatives



They use one cube to represent one metre. They could use a scale, e.g., 1 cube represents 9 metres, or 1 cube represents 3 metres. Students multiply the horizontal cubes and vertical cubes by the same factor to keep the proportion the same.

This would result in something like this:



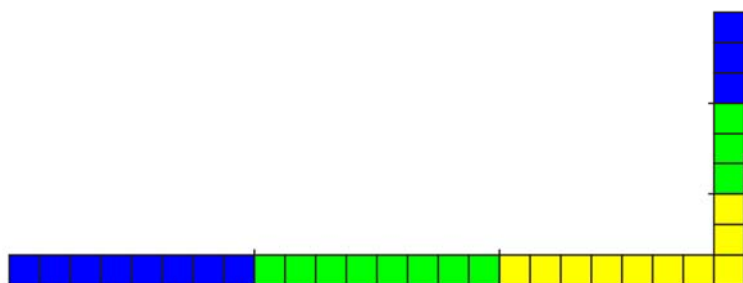
1 cube = 9 m

If each cube represents 9 m, then the maximum height of the library would be: 3 cubes \times 9 m = 27 m

- 2.

Problem-Solving Strategies:

- Draw a scale diagram



1 cube = 3 m

If each cube represents 3 m, then the maximum height of the library would be: 9 cubes \times 3 m = 27 m

Note: Students are focussed on the edge of the cubes.

Grade 7

Sample Solutions

3. a)
$$\frac{\text{height}}{\text{shadow}}$$
$$\frac{3\text{m}}{8\text{m}} = \frac{x}{72}$$

Problem-Solving Strategies:

- Use proportion (equivalent ratios)
- Use “within” and “between” ratios

They would examine the denominators and determine what factor 8 was multiplied by to get 72. ($72 \div 8 = 9$) They multiply the numerator by the same factor. ($3 \times 9 = 27$ metres)

b)
$$\frac{\text{height}}{\text{height}}$$
$$\frac{3\text{m}}{x\text{m}} = \frac{8}{12} = \frac{1}{9}$$

since $\frac{3}{27} = \frac{1}{9}$

Therefore, maximum height is 27 metres.

Grade 8

Sample Solutions

Students’ solutions could include any of the Grade 7 answers.

1.
$$\frac{\text{height}}{\text{shadow}}$$
$$\frac{3\text{m}}{8\text{m}} = \frac{x}{72}$$

Problem-Solving Strategies:

- Use proportion (equivalent ratio)
- Use “within” and “between” ratios

a) Students look at the known proportion. They look at what factor 3 was multiplied by to get 8. ($8 \div 3 =$ (approximately) 2.66667) They divide 72 by the same factor. ($72 \div 2.66667 = 27$ metres)

b) Students could use the proportion $\frac{3}{8}$.

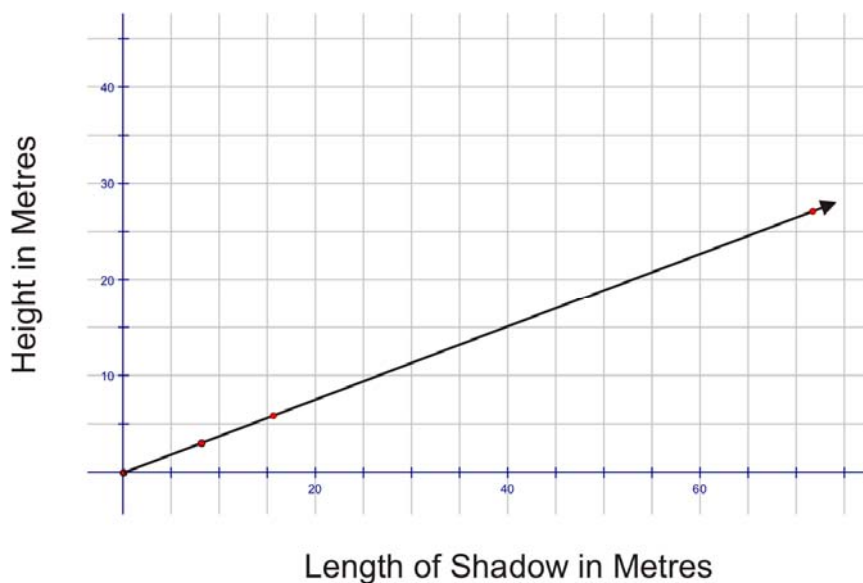
$$\frac{3}{8} \text{ of } 72 = 27 \text{ metres}$$

Students' solutions could include any of the Grade 7 answers.

2. Students graph 8 across and 3 up (8, 3) and then continue to plot points that translate $[+8, +3]$. This would lead them to (72, 27). When the horizontal distance is 72 m, the height is at 27 m.

Problem-Solving Strategies:

- Use a pattern to draw a graph



3.

Height of Building (m)	Length of Shadow (m)	Reason
3	8	
6	16	double
12	32	double
24	64	double
27	72	add original

Problem-Solving Strategies:

- Use a ratio table

Students' solutions could include any of the Grade 7 and 8 answers.

a) “Within” ratios

Problem-Solving Strategies:

- Use “within” and “between” ratios

$$\frac{\text{pole height}}{\text{pole shadow}} = \frac{\text{building height}}{\text{building shadow}}$$

$$\frac{3}{8} = \frac{x}{27}$$

Multiply both sides by 72.

$$(72)\left(\frac{3}{8}\right) = x\left(\frac{72}{72}\right)$$

$$27 = x$$

The building can be 27 metres tall.

b) “Between” ratios

$$\frac{\text{pole height}}{\text{building height}} = \frac{\text{pole shadow}}{\text{building shadow}}$$

$$\frac{3}{x} = \frac{8}{72}$$

$$8x = (3)(72)$$

$$\begin{aligned} x &= (3)(72) \div 8 \\ &= 27 \end{aligned}$$

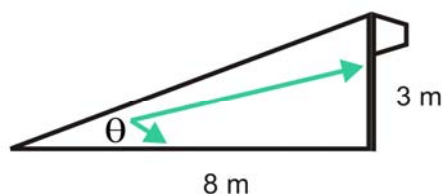
The building can be 27 metres tall.

Students' solutions could include any of the Grades 7, 8, and 9 answers.

1. Connect the angle to known information.

Problem-Solving Strategies:

- Use trigonometry



$$\tan \theta = \frac{3}{8}$$

$$\theta = 20.55604522 \text{ degrees}$$

Similarly with the building,

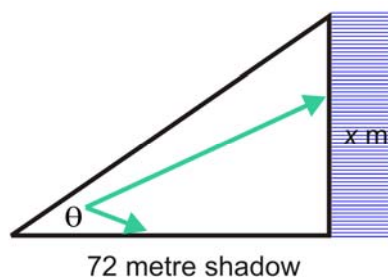
$$\tan 20.55604522 = \frac{x}{72}$$

$$x = 72 \tan (20.55604522)$$

$$= 27$$

$$x = 27 \text{ m}$$

The building can be no more than 27 metres tall.



2. a) “Within” ratios

$$\frac{x}{72} = \frac{3}{8}$$

$$x = \frac{3}{8}(72)$$

$$= 27$$

Problem-Solving Strategies:

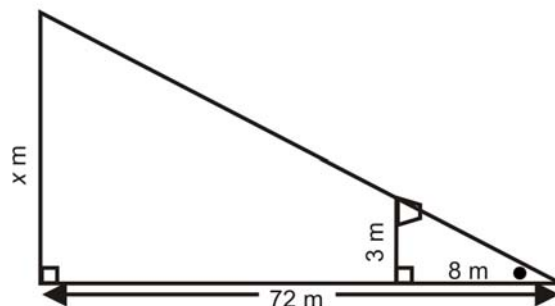
- Use similar triangles
- Use “within” and “between” ratios

- b) “Between” ratios

$$\frac{x}{3} = \frac{72}{8}$$

$$x = \frac{72}{8}(3)$$

$$= 27$$



Therefore, the building should have a maximum height of 27 metres.

Is This Always True?

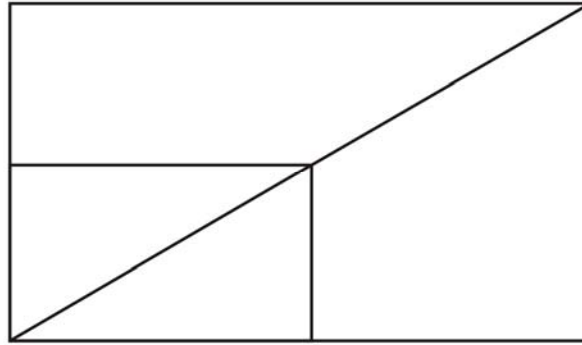
(Reflecting, Reasoning and Proving)

Grades 7–10

Name:

Date:

Is it always true that if two rectangles are similar their diagonals will match up when they are stacked one on top of the other. See the diagram.



Grade 7

Yes. I cut several rectangles from graph paper that are similar, glued them on top of each other and used a ruler to show that their diagonals line up.

Sample Solutions

Problem-Solving Strategies:

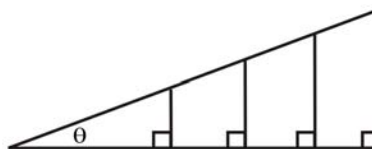
- Make paper models
- Use logical reasoning

Grade 8

Yes. I cut several similar triangles starting with similar rectangles and cutting along the diagonal. Using the reasoning that the angles in the similar triangle are the same and the sides are proportional, the diagonals show that the hypotenuse of triangles extend along one line. Since the hypotenuse is the diagonal, the property is true.

Problem-Solving Strategies:

- Use similar triangles



Grade 9

Sample Solutions

Yes. I measured the sides of the rectangle to find the slope of the diagonal in each case. Since the slopes are all the same, the property is always true.

Problem-Solving Strategies:

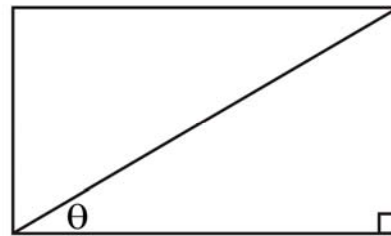
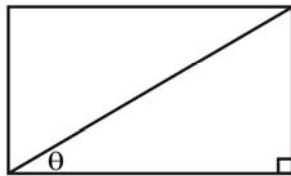
- Create similar rectangles on graph paper
- Compare slopes

Grade 10

Yes. I constructed similar rectangles and used trigonometry to find the angle between the length and the diagonal of each triangle within the rectangles. Since the angle will be the same in each similar rectangle, the property is true.

Problem-Solving Strategies:

- Use trigonometry



Is This Always True?

Grades 7–10

Name:

Date:

Sarah claims that when two triangles have one angle the same size then the triangles have proportional sides?

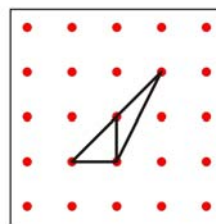
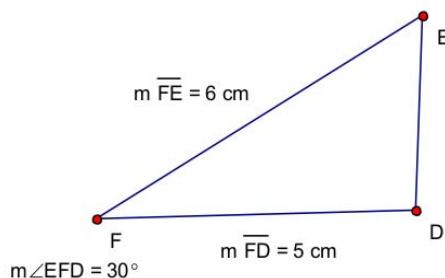
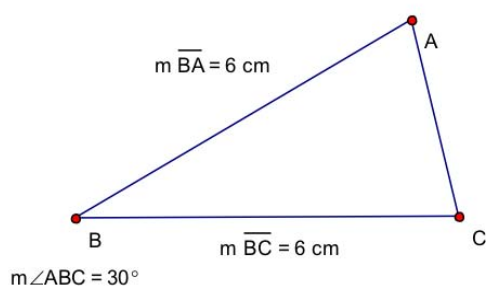
Grades 7–10

Sample Solutions

No. Use graph paper, a protractor, and a ruler to draw several triangles that have one angle the same but the sides are not proportional. Construct an example, using geoboards or The Geometer's Sketchpad®4.

Problem-Solving Strategies:

- Construct counter-example
- Use a manipulative
- Use technology (GSP®4)



The ratio of side AB to side EF is $6:6 = 1.1$
The ratio of side BC to side FD = $6:5$ (not 1:1)

Grades 8–10

No. All three sides of two triangles are proportional only if they are similar triangles. The condition for similar triangles is that all three angles of one triangle would be the same as the three angles in the other triangle.

Problem-Solving Strategies:

- Use reasoning

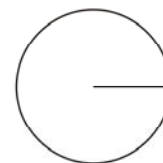
Is This Always True?

Grades 7–10

Name:

Date:

Is it always true that the circumference of a circle divided by the radius will always give the same answer?



Grades 7–10

Sample Solutions

Yes.

- Students could choose some examples of circles of different sizes, and measure their circumferences and radii. They divide the radius and verify that it is the same.
- Students could construct several circles, using a compass. They measure the perimeter and the diameter with a piece of string, and use a calculator to divide the circumference by the radius for each example to show that the answer is the same for each case.

Problem-Solving Strategies:

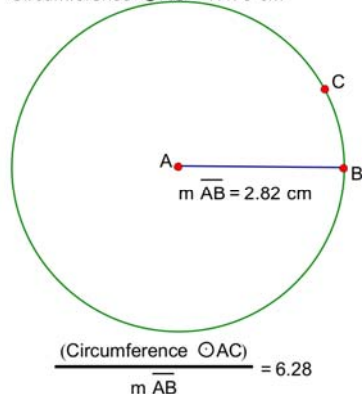
- Construct examples, using GSP®4 or a compass
- Use a formula

Radius (r)	Circumference (c)	$\frac{c}{r}$
5	10π	$\frac{10\pi}{5} = 2\pi$
20	40π	$\frac{40\pi}{20} = 2\pi$

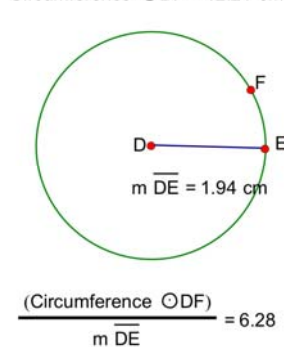
The answer will always be 2π .

- Students could use GSP®4 to create a circle, and measure the radii and circumference of each circle. They calculate the ratio circumference divided by the radii and tabulate or drag to calculate a large number of examples.

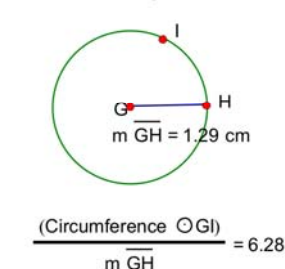
Circumference $\odot AC = 17.70$ cm



Circumference $\odot DF = 12.21$ cm



Circumference $\odot GI = 8.11$ cm



- Students could use the formula.

$$\frac{2\pi r}{r} = 2\pi$$